

# Dense Matter and Functional Renormalization Group

Matthias Drews  
Thomas Hell, Bertram Klein, Wolfram Weise

Technische Universität München, T39

05.06.2012



# Content

- 1 The Nucleon-Meson Model
- 2 Neutron stars and neutron matter
- 3 Summary

# Chemical Freeze-Out and QCD phase transition

**Conjecture** (for low baryon densities):

only multi-particle processes/collective effects can maintain chemical equilibrium

$$\Rightarrow T_{\text{chemical freeze-out}} \simeq T_{\text{chiral crossover}}.$$

Braun-Munzinger, Stachel, Wetterich, Phys.Lett.B596, 2004

# Chemical Freeze-Out and QCD phase transition

**Conjecture** (for low baryon densities):

only multi-particle processes/collective effects can maintain chemical equilibrium

$$\Rightarrow T_{\text{chemical freeze-out}} \simeq T_{\text{chiral crossover}}.$$

Braun-Munzinger, Stachel, Wetterich, Phys.Lett.B596, 2004

**Question:** What happens at large densities?

large  $\mu$  is territory of nuclear physics  $\rightarrow$  effective **nucleon-meson model**  
applicable

Floerchinger, Wetterich, arXiv:1202.1671

## Lagrangian:

$$\begin{aligned}\mathcal{L} = & \bar{\psi} \left( i\cancel{d} + g_\omega \cancel{\omega} + \mu \gamma^0 + g_\pi (\sigma + i\gamma^5 \cancel{\pi} \cdot \boldsymbol{\tau}) \right) \psi + \\ & + \frac{1}{2}(\partial\sigma)^2 + \frac{1}{2}(\partial\boldsymbol{\pi})^2 + \partial_{[\mu} \omega_{\nu]} \partial^{[\mu} \omega^{\nu]} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + U_{\text{mic}}(\sigma, \boldsymbol{\pi}).\end{aligned}$$

contains protons and neutrons, omega-mesons, sigmas and pions.

# Mean-Field Approximation

Integrate out nucleons;  $\sigma$  and  $\omega_0$  acquire expectation values.  
Effective potential:

$$U = U_{\text{vac}}(\sigma, \pi, \omega_0) - 4P_{\text{free gas}}(T, \mu, \sigma, \omega_0),$$
$$P_{\text{free gas}} = \int \frac{d^3 p}{(2\pi)^3} \log \left[ 1 + e^{-\beta(\sqrt{p^2+m^2}-\mu_{\text{eff}})} \right] + (\mu_{\text{eff}} \rightarrow -\mu_{\text{eff}}),$$
$$m = g_\pi \sigma, \quad \mu_{\text{eff}} = \mu + g_\omega \omega_0.$$

# Mean-Field Approximation

Integrate out nucleons;  $\sigma$  and  $\omega_0$  acquire expectation values.  
Effective potential:

$$U = U_{\text{vac}}(\sigma, \pi, \omega_0) - 4P_{\text{free gas}}(T, \mu, \sigma, \omega_0),$$
$$P_{\text{free gas}} = \int \frac{d^3 p}{(2\pi)^3} \log \left[ 1 + e^{-\beta(\sqrt{p^2+m^2}-\mu_{\text{eff}})} \right] + (\mu_{\text{eff}} \rightarrow -\mu_{\text{eff}}),$$
$$m = g_\pi \sigma, \quad \mu_{\text{eff}} = \mu + g_\omega \omega_0.$$

$U_{\text{vac}}$ : expand around **liquid-gas phase transition** ( $T = 0, \mu = \mu_c$ ).

$$U_{\text{vac}} = -m_\pi^2 f_\pi (\sigma - f_\pi) + \sum_{n=1}^{N_{\text{max}}} a_n (\rho - \rho_0)^n - \frac{m_\omega^2}{2} \omega_0^2, \quad \rho = \frac{1}{2} \sigma^2 + \frac{1}{2} \pi^2.$$

Resulting values:

sigma mass  $m_\sigma = 670$  MeV, (exp:  $484 \pm 17$  MeV),  
compressibility  $K = 300$  MeV, (exp:  $240 \pm 30$  MeV),  
surface tension  $\Sigma = 42000$  MeV, (exp:  $42200$  MeV).

### Procedure:

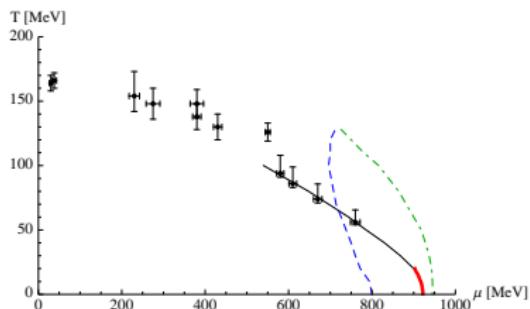
- ➊ Solve simultaneously:

$$\partial_\sigma U = 0, \quad \partial_{\omega_0} U = 0.$$

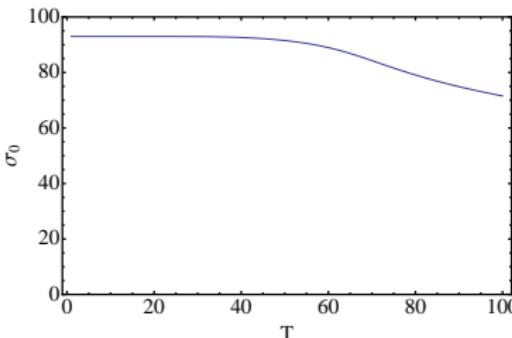
- ➋ Compute e.g. density:  $n_B = \partial_\mu U$ .

# Results

No chiral phase transition at chemical freeze-out:



(a) Chemical freeze-out.



(b)  $\mu = 750$  MeV.

Floerchinger, Wetterich, arXiv:1202.1671

Andronic, Braun-Munzinger, Stachel, Phys.Lett.B673, 2009

# Neutron Stars

Study **neutron star matter**. Additional ingredients:

- ❶ Different **chemical potentials**  $\mu_p, \mu_n$  for neutrons and protons.
- ❷ **Electrons** with chemical potential  $\mu_e$ .
- ❸  $\rho$  **degree of freedom**.

New Lagrangian:

$$\begin{aligned}\mathcal{L} = & \bar{\psi} \left( i\cancel{d} + g_\omega (\cancel{\psi} + \cancel{\rho} \cdot \boldsymbol{\tau}) + g_\sigma (\cancel{\sigma} + i\gamma_5 \cancel{\pi} \cdot \boldsymbol{\tau}) + (\mu_p - \mu_n) \gamma^0 \right) \psi + \\ & + \bar{\psi}_e (i\cancel{d} + \mu_e \gamma^0) \psi_e + \frac{1}{2} (\partial \sigma)^2 + \frac{1}{2} (\partial \pi)^2 + U(\rho, \sigma) + \\ & + \partial_{[\mu} \omega_{\nu]} \partial^{[\mu} \omega^{\nu]} + \partial_{[\mu} \rho_{\nu]} \partial^{[\mu} \rho^{\nu]} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{2} m_\rho^2 \rho_\mu \rho^\mu.\end{aligned}$$

Now  $\sigma, \omega_0$  and  $\rho_0^3$  get expectation values.

# Neutron Stars

Study **neutron star matter**. Additional ingredients:

- ➊ Different **chemical potentials**  $\mu_p, \mu_n$  for neutrons and protons.
- ➋ **Electrons** with chemical potential  $\mu_e$ .
- ➌  $\rho$  **degree of freedom**.

New Lagrangian:

$$\begin{aligned} \mathcal{L} = & \bar{\psi} \left( i\cancel{D} + g_\omega (\cancel{\psi} + \cancel{\rho} \cdot \boldsymbol{\tau}) + g_\sigma (\cancel{\sigma} + i\gamma_5 \cancel{\pi} \cdot \boldsymbol{\tau}) + (\mu_p - \mu_n) \gamma^0 \right) \psi + \\ & + \bar{\psi}_e (i\cancel{D} + \mu_e \gamma^0) \psi_e + \frac{1}{2} (\partial \sigma)^2 + \frac{1}{2} (\partial \pi)^2 + U(\rho, \sigma) + \\ & + \partial_{[\mu} \omega_{\nu]} \partial^{[\mu} \omega^{\nu]} + \partial_{[\mu} \rho_{\nu]} \partial^{[\mu} \rho^{\nu]} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{2} m_\rho^2 \rho_\mu \rho^\mu. \end{aligned}$$

Now  $\sigma, \omega_0$  and  $\rho_0^3$  get expectation values.

New **effective chemical potential**:

$$\mu_{\text{eff},p} = \mu + g_\omega (\omega_0 + \rho_0^3),$$

$$\mu_{\text{eff},n} = \mu + g_\omega (\omega_0 - \rho_0^3).$$

Solve simultaneously:

$$\partial_\sigma U = 0, \quad \partial_{\omega_0} U = 0, \quad \partial_{\rho_0^3} U = 0,$$

$\mu_n = \mu_p + \mu_e$  **beta equilibrium**,

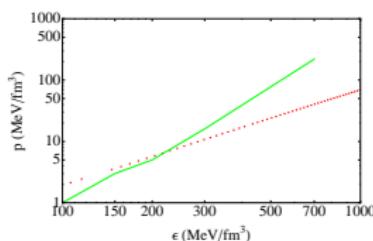
$n_p = n_e$  **charge neutrality**,

with  $\mu_e = \sqrt{p_F^2 + m_e^2}$  and  $p_F^3 = 3\pi^2 n_e$ .

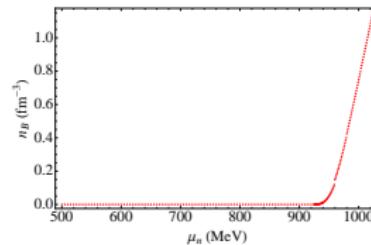
**Comparison:**

Red: Neutron Star Matter, Green: Akmal et al.

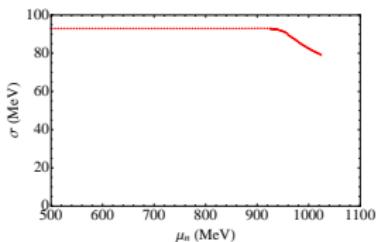
Akmal, Pandharipande, Ravenhall, Phys.Rev.C58, 1998



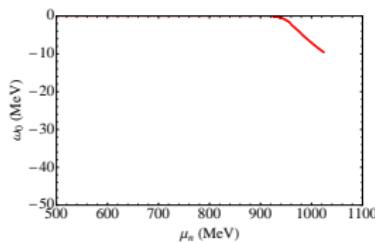
(a) Equation of state.



(b) Baryon density.



(c) Chiral order parameter.

(d)  $\omega_0$ -expectation value.

# Beyond Mean Field: Functional Renormalization Group

## Wetterich's Flow Equation

$$\begin{aligned}\partial_t \Gamma_k[\Phi_k] &= \frac{1}{2} \text{Tr} \partial_t R_k \left( \Gamma_k^{(2)}[\Phi_k] + R_k \right)^{-1} = \\ &= \frac{1}{2} \otimes \circlearrowleft .\end{aligned}$$

Wetterich, Phys.Lett.B301, 1993

$k$ : **renormalization scale**,  $t = \log k/\Lambda$ .

$\Gamma_k$ :  $k$ -dependent **Effective Action**,  $\Gamma_k^{(2)}$ : its second derivative with respect to the fields.

$R_k$ : **regulator function** to cut off low momenta.

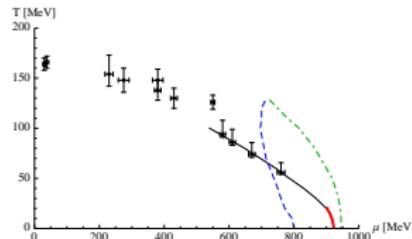
→ Interpolation between **UV-action** at  $k = \Lambda_{\text{UV}}$  and **full effective action** at  $k = 0$ .

Flow Equation:

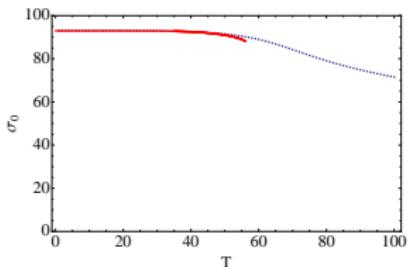
Truncated Wetterich equation for the QM-model

$$\begin{aligned}\partial_t \Gamma_k = \beta V \frac{k^5}{12\pi^2} & \left[ 3 \frac{1 + 2 \frac{1}{e^{\beta E_\pi} - 1}}{E_\pi} + \frac{1 + 2 \frac{1}{e^{\beta E_\sigma} - 1}}{E_\sigma} - \right. \\ & \left. - \frac{4 \cdot 2}{E_q} \left( 1 - \frac{1}{e^{\beta(E_q - \mu_{\text{eff}})} + 1} - \frac{1}{e^{\beta(E_q + \mu_{\text{eff}})} + 1} \right) \right].\end{aligned}$$

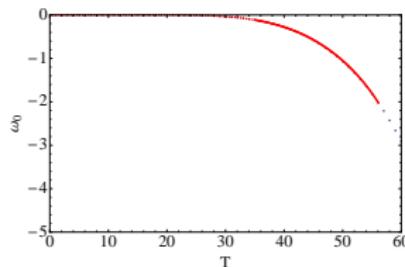
# Preliminary results



(a) Chemical freeze-out.



(b) Chiral order parameter.



(c)  $\omega_0$  expectation value.

$\mu = 750$  MeV. Red: FRG, black: mean-field.

# Summary and Outlook

- ① A **Nucleon-meson model** fitted to the liquid-gas phase transition shows **no** indication of **first order phase transition** at chemical freeze out.
- ② Neutron star matter may be studied via implementing **beta equilibrium**.
- ③ Excitations beyond mean field: **Functional Renormalization Group**.